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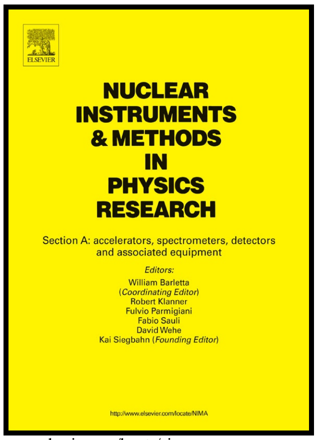

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Estimating the Effective System Dead Time Parameter for Correlated Neutron Counting

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Abstract

Neutron time correlation analysis is one of the main technical nuclear safeguards techniques used to verify declarations of, or to independently assay, special nuclear materials. Quantitative information is generally extracted from the neutron-event pulse train, collected from moderated assemblies of ^{3}He proportional counters, in the form of correlated count rates that are derived from event-triggered coincidence gates. These count rates, most commonly referred to as singles, doubles and triples rates etc., when extracted using shift-register autocorrelation logic, are related to the reduced factorial moments of the time correlated clusters of neutrons emerging from the measurement items. Correcting these various rates for dead time losses has received considerable attention recently. The dead time losses for the higher moments in particular, and especially for large mass (high rate and highly multiplying) items, can be significant. Consequently, even in thoughtfully designed systems, accurate dead time treatments are needed if biased mass determinations are to be avoided. In support of this effort, in this paper we discuss a new approach to experimentally estimate the effective system dead time of neutron coincidence counting systems. It involves counting a random neutron source (e.g. AmLi is a good approximation to a source without correlated emission) and relating the second and higher moments of the neutron number distribution recorded in random triggered interrogation coincidence gates to the effective value of dead time parameter. We develop the theoretical basis of the method and apply it to the Oak Ridge Large Volume Active Well Coincidence Counter using sealed AmLi radionuclide neutron sources and standard multiplicity shift register electronics. The method is simple to apply compared to the predominant present approach which involves using a set of ^{252}Cf sources of wide emission rate, it gives excellent precision in a conveniently short time, and it yields consistent results as a function of the order of the moment used to extract the dead time parameter. This latter observation is reassuring in that it suggests the assumptions underpinning the theoretical analysis are fit for practical application purposes. However, we found that the effective dead time parameter obtained is not constant, as might be expected for a parameter that in the dead time model is characteristic of the detector system, but rather, varies systematically with gate width.

Keywords: neutron coincidence counting; multiplicity counting; dead time correction; fissile material assay

Introduction

Event triggered neutron time correlation analysis is extensively used to quantify fissioning systems [1, 2]. Examples include the passive assay of Pu items in terms of the equivalent ^{240}Pu spontaneous fission rate, the measurement of U items from the induced fission strength resulting from an external interrogating source, and the analysis of sub-critical assemblies. In all cases the use of correlated analysis is used to provide more information about the measured items than gross counting alone can do – especially because the gross counting rate depends on non-fission processes such as (α,n) production. An important aspect of neutron time correlation is how to correct the various observed rates of interest for dead time losses [3-5]. There are two considerations: first having a formalism that describes the behavior, and second, having an experimental way to determine the dead time parameter. In this work, we adopt the formalism (mathematical dead time model) of Hauck et al. [6], and develop a new approach to extracting the effective model dead time parameter under the assumptions of that model. The concept of an effective dead time parameter will be shown by direct experiment to be only an approximation when applied to a representative measurement instrument. The model error in the dead time parameter determination from the count distribution evidently far exceeds the readily achieved measurement precision reported. However, this does not preclude the application of the model with suitable parameters to problems of practical interest. In our proposed new approach the neutron count (number) distribution is measured using a time-random neutron source placed in the neutron detector cavity while sampling the detected-neutron pulse train a large number of times by random placement of counting gates. The use of a (single) random (uncorrelated) source may be seen as an advantage, when such a source is available, over using a collection of ^{252}Cf sources of widely different emission rates. In our case we used AmLi sources which we hold as active neutron interrogation sources. They have a long working life because the half-life of ^{241}Am is over 400 years, and so unlike ^{252}Cf, they do not require regular replacement. The observed variance of the recorded neutron number distribution from a random source is expected to be narrower than for an ideal Poisson distribution, because of dead time losses, which introduces a degree of correlation between closely spaced events. Menaa et al [7] successfully exploited this idea to obtain experimental estimates of the effective system dead times of neutron multiplicity counters using sealed AmLi radionuclide sources. The basis of their analysis technique was the theoretical result of Foglio Para and Bettoni [8] for the number distribution recorded in the ideal paralyzable (Type I or extending) dead time model for the 'not free' single-chain counter case. By applying the method of Laplace transforms to the theoretical number distribution, an expression relating the variance, σ_i^2, of the i-distribution to its mean, $\langle i \rangle$, and the ratio, φ, of the dead time parameter, δ, to the gate width, T_g can be derived. The result is [7]:

$$\sigma_i^2 = \langle i \rangle \left(1 - \varphi\left(2 - \varphi\right)\langle i \rangle\right)$$

In the small φ limit ($0 < \phi \ll 1$) we can linearize the expression to obtain:

$$\varphi = \frac{\delta}{T_g} \approx \frac{1}{2\langle i \rangle}\left(1 - \frac{\sigma_i^2}{\langle i \rangle}\right)$$

We see that as φ approaches zero from above then σ_i^2 approaches $\langle i \rangle$, which is the well-known result we would expect for a pure Poisson process.

Solving the full quadratic expression, $\varphi^2 - 2\varphi + \frac{\langle i \rangle - \sigma_i^2}{\langle i \rangle^2} = 0$, for φ we obtain the following physically meaningful solution:

$$\varphi = 1 - \sqrt{1 - \left[\frac{\langle i \rangle - \sigma_i^2}{\langle i \rangle^2}\right]} = 1 - \sqrt{1 - \left[\frac{1 - \sigma_i^2/\langle i \rangle}{\langle i \rangle}\right]}$$

Because of deadtime, the ratio $\sigma_i^2/\langle i \rangle$ is expected to be slightly less than unity for the counters of interest to our discussion, and so expanding the square root using the Binomial theorem we immediately see using the form of the solution given on the right hand side is consistent with the linearized solution given previously. The other mathematical solution, $\varphi = 1 + \sqrt{1 - \left[\frac{\langle i \rangle - \sigma_i^2}{\langle i \rangle^2}\right]}$ is not physically appropriate.

The estimate for the mean number of counts in the gate, μ_i, expected in the absence of dead time losses may then be obtained by solving the familiar transcendental equation $\langle i \rangle = \mu_i e^{-\mu_i \varphi}$, where e is Euler's number, the base of the natural logarithm [9].

In an earlier work to that of Menaa et al [7], Robba, Dowdy and Atwood (RDA) [10] advocated a similar approach to practical effective dead time parameter estimation for neutron time correlation counting. Their starting point was the first order derivation of the Feynman-Y excess variance statistic [2] influenced by dead time. For a non-multiplying random neutron source Equation (11) of RDA [10] may be written as follow:

$$Y + 2\varphi\langle i \rangle = 0$$

where

$$Y = \frac{\sigma_i^2}{\langle i \rangle} - 1$$

Substituting and re-arranging we obtain the same approximate result as before, namely:

$$\varphi = \frac{\delta}{T_g} \approx \frac{1}{2\langle i \rangle}\left(1 - \frac{\sigma_i^2}{\langle i \rangle}\right)$$

Note that Equation (9) in RDA [10] should also give this result if it were not for what appears to be a simple sign error.

More recently, a comprehensive forward-predictive theoretical development of various time correlation counting rates, including the distortion due to a dead time, has been published by Hauck et al [6]. The results are exact within the limitations of the dead time model, which assumes a single-chain detector, with a single exponential time constant or dieaway time, subject to a fixed paralyzable dead time period following each neutron detection-event. This physical picture is simple - although the mathematical development quickly becomes tedious. We may (correctly) imagine the information about the origin of a neutron inside the item being encoded in the frequency at which neutrons emerge from the item in clusters. That is the rate at which clusters of one, two, three, four … neutrons, with a common ancestry (e.g. an (α,n) of spontaneous fission initiated fission chain), emerge from the item and may strike the detector. These clusters, or bursts, of neutrons are then slowed down and thermalize in the detector. Each thermal neutron has the same probability as the rest of the cluster of being lost or detected by the system. This results in both a softer detected number multiplicity distribution than emerges from the item, and the clusters being spread out in time according to the dieaway time of the detector. The theoretical development of a dead time correction starts with this simple point-process statistical model and overlays, onto the average temporal behavior of potentially overlapping detected clusters, a system dead period after each detection interaction. The resulting dead time correction for the various rates depends on the degree of correlation on the pulse train- in other words, on the dead time corrected correlated rates- because the dead time free correlated rates provide a mathematical basis set for representing the statistical properties of the neutron source term. However, for a random neutron source the dead time model predictions simplify considerably because there are no sources of two, three, or higher neutrons. With this in mind we can therefore adapt the theoretical results of Hauck et al, for the special case of a random neutron source, so as to obtain the following relations for the expected average and expected reduced second factorial moment of the number distribution, which permit the corresponding dead time model parameter to be estimated experimentally:

$$\langle i \rangle = \int_0^{T_g} dt_1 d_1 \, e^{-d_1 \delta} = T_g d_1 e^{-d_1 \delta}$$

$$\frac{\langle i(i-1) \rangle}{2} = \int_0^{T_g - \delta} dt_1 \int_{t_1+\delta}^{T_g} dt_2 \, d_1^2 e^{-2d_1 \delta} = \frac{\left(T_g - \delta\right)^2}{2} d_1^2 e^{-2d_1 \delta} = \frac{\left(T_g - \delta\right)^2}{2} \frac{\langle i \rangle^2}{T_g^2}$$

In these equations we are using the bra-ket $\langle \quad \rangle$ notation to denote the i-distribution expectation or averaged value of the enclosed quantity; in this case $\frac{\langle i(i-1) \rangle}{2}$ is also referred to as the reduced second factorial moment. The far right hand form of the second expression follows by substituting from the first. d_1 is the singles (gross or totals) rate of events that would be recorded in the absence of dead time. It is the average rate at which neutrons are interacting in the detector medium. T_g is the width or duration of the time gate. To clarify these definitions let $S_m = \langle i \rangle / T_g$ represent the measured (singles) count rate and $S_c = d_1$ the dead time corrected rate. Then, $S_m = S_c e^{-S_c \delta} = S_c e^{-(S_c T_g)\phi}$, which we recognize as the earlier transcendental form for paralyzable dead time and a Poisson source.

Re-arranging the two relationships we obtain the following expression for φ:

$$\varphi = 1 - \sqrt{\frac{\langle i(i-1) \rangle}{\langle i \rangle^2}} = 1 - \sqrt{1 - \left[\frac{\langle i \rangle - \sigma_i^2}{\langle i \rangle^2}\right]}$$

where, in order to obtain the far right hand side form, we have used the following equality of statistical expectation values:

$$\langle i(i-1)\rangle = (\langle i^2\rangle - \langle i\rangle^2) + \langle i\rangle^2 - \langle i\rangle = \sigma_i^2 + \langle i\rangle^2 - \langle i\rangle$$

Note the *exact* expression for φ obtained using the theory of Hauck et al [6] agrees with the quadratic solution developed by Menaa et al [7] which was founded on the work of Foglio Para and Bettoni [8]. We shall therefore adopt this result (rather than the RDA [10] approximation) in the analysis of the experimental data to be discussed later.

The value of the dead time parameter follows from the definition of ϕ:

$$\delta = \phi T_g$$

Importantly, in addition, the formalism of Hauck et al [6] readily lends itself to the derivation of expressions for higher-order statistics of the observed count distribution when a random neutron source is present. Thus, for example, we have for the third reduced factorial moment:

$$\frac{\langle i(i-1)(i-2)\rangle}{6} = \int_0^{T_g-2\delta} dt_1 \int_{t_1+\delta}^{T_g-\delta} dt_2 \int_{t_2+\delta}^{T_g} dt_3\, d_1^3 e^{-3d_1\delta} = \frac{\left(T_g-2\delta\right)^3}{6} d_1^3 e^{-3d_1\delta}$$

or

$$\frac{\langle i(i-1)(i-2)\rangle}{\langle i\rangle^3} = \left(1 - 2\frac{\delta}{T_g}\right)^3$$

from which we obtain an alternative expression that can also potentially be used to experimentally estimate a value for φ, namely:

$$\varphi = \frac{1}{2}\left[1 - \sqrt[3]{\frac{\langle i(i-1)(i-2)\rangle}{\langle i\rangle^3}}\right]$$

Extending to the fourth order reduced factorial moment we find:

$$\frac{\langle i(i-1)(i-2)(i-3)\rangle}{24} = \int_0^{T_g-3\delta} dt_1 \int_{t_1+\delta}^{T_g-2\delta} dt_2 \int_{t_2+\delta}^{T_g-\delta} dt_3 \int_{t_3+\delta}^{T_g} dt_4\, d_1^4 e^{-4d_1\delta} = \frac{\left(T_g-3\delta\right)^4}{24} d_1^4 e^{-4d_1\delta}$$

from which we obtain:

$$\varphi = \frac{1}{3}\left[1 - \sqrt[4]{\frac{\langle i(i-1)(i-2)(i-3)\rangle}{\langle i\rangle^4}}\right]$$

These cases are sufficient to establish the general pattern by induction. Out of interest, and because it can be useful in checking practical implementations of these results, we also see that to first order, that is in the small dead time limit, the corresponding dead time correction factors for the first few reduced factorial moments for a random neutron source scale as: $(1+\delta)$, $(1+4\delta)$, $(1+9\delta)$, $(1+16\delta)$, $(1+25\delta)$, etc. When the pulse train is only weakly correlated (low detector efficiency and short fission chain items) and the dead time correction is small, these simple results are useful approximations and we have used them in sensitivity and uncertainty analysis for multiplicity counting – although they must be treated with care and are not recommended for practical use outside of this very narrow setting.

Experimental Demonstration

In order to study how the proposed new approach to experimentally determining the effective system dead time from the recorded number distribution works in practice, data were collected using the Oak Ridge National Laboratory's Large Volume Active Well Coincidence Counter (ORNL LV-AWCC). The LV-AWCC is a thermal-well counter similar in design to the standard AWCC (Canberra Industries Inc., model JCC-51 [11]) but it has been scaled to a larger cavity diameter and it uses a higher ^{3}He fill pressure. There are 48 ^{3}He-filled cylindrical proportional counters, of 1 inch external diameter at a partial fill pressure of 4.5 atm., arranged in two concentric rings about an 11 inch diameter, 15 inch tall assay measurement cavity. For these measurements the Cd-liner was in place and the largest cavity configuration was used. The LV-AWCC is a general purpose instrument which may be used to assay uranium by inducing fission by placing an AmLi source in polyethylene end plugs. But routinely the system is also used with graphite end-plugs (without the AmLi sources) to perform passive assay of spontaneously fissioning items including multiplicity (singles, doubles, triples) mode. In that sense it is a representative of a general purpose counter with a wide range of measurement applications.
The proportional counters are hard wired into eight groups of six with each group being serviced by a separate Canberra JAB-01 Amptek A-111 based amplifier/discriminator unit. In this way, the inner ring of proportional counters is catered for by four amplifiers, as too is the outer ring. The 8 streams of 52 ns wide TTL logic pulses are electronically summed via a multi-input derandomizer circuit and fed as a single pulse train into the shift register. Leading edge pulses arriving close in time at the input of the derandomizer remain distinct because, in such cases, the queue of events are placed on the output pulse train on the next free clock cycle. The shift register also has a 16 pulse deep input buffer (which should be redundant given the action of the derandomizer) and so is not expected to introduce any further dead time loss. A photograph of the system is shown in Figure 1. To represent a random neutron source, a pair of AmLi neutron sources, normally located in the high-density polyethylene end-plugs for active interrogation, were positioned side-by-side inside the measurement cavity on either side of the cylindrical

axis. These sources are of type Gammatron model AN-HP, Serial numbers N-458 and N-459, each containing 1.20 Ci (± 1%) of ^{241}Am, and having a measured neutron emission rate of $4.9x10^4$ $n.s^{-1}$ (± 3 %)) Each neutron source is doubly encapsulated in stainless steel with external dimensions of roughly 25 mm diameter by 35 mm long. Each is additionally shielded by an external tungsten 'pot' with a minimum wall thickness of >2.5 mm to reduce the dose from the ^{241}Am 60 keV gamma-ray emission when handling the sources.

Figure 1. A photograph of the ORNL LV-AWCC. The top-plug is shown removed and is resting on the electronics enclosure.

Data was acquired using a hand-held multiplicity shift register [12]. Standard operational settings were used, in particular the high-voltage of 1700 V, is slightly above the knee of the coincidence (doubles) rate plateau measured using a ^{252}Cf reference source, and an event-triggered-gate predelay [1] of 4.5 μs was used. The long delay [1] is fixed in the electronics hardware at 4096 μs. With a coincidence gate width of 64 μs, the ambient (room) singles, doubles, and triples background rates were approximately 8.32 counts per second (cps), 0.123 doubles per second (dps), and 0.022 triples per second (tps), respectively. No background subtraction is needed, however, because the singles rate simply contributes a small additional random contribution to the AmLi rate and the ambient correlated rates are negligible compared to the uncertainties. For reference, the count rate with the AmLi sources present is about 37 kcps.

The shift register data is in the form of two histograms: the (R+A)-histogram and the A-histogram [1,2]. The (R+A)-histogram is count-distribution-generated by opening a gate after a predelay period for each incoming event. The A-histogram is the count-distribution-generated by opening a gate after the long delay. Provided the quiescent behavior of the detector system is reestablished in a time shorter than the predelay following each event, then the two number distributions should be equivalent within their combined statistical sampling uncertainties. We can test for this by forming the bias factor defined by:

$$Bias = 100\left[\frac{\langle i\rangle_{R+A}}{\langle i\rangle_{A}} - 1\right], \%$$

where $\langle i\rangle_{R+A}$ and $\langle i\rangle_{A}$ are the average number of counts per gate in the (R+A)- and A-histograms respectively.

Measurements were performed as a function of gate width for settings of 2, 4, 8, 16, 32, 64, 128 and 256 μs. For each gate width setting, data were acquired for 24 counting periods each of 300 sec duration. This allowed a statistical analysis to be performed on the sequence of 24 cycles for the assignment of experimental precision uncertainty in the dead time parameter estimates. For each run, the (R+A)- and A-histogram data were analyzed separately, checked for bias and self-consistency, and then combined into a single 48 cycle data set.

Results

The dead time parameter was determined using this analysis with the neutron count distribution of each of these 24 cycles in the (R+A) and A gates, in addition to the combined gate. Table 1 reports the values of this analysis.

Table 1. Results of dead time analysis for (R+A)- and A-gates, including bias

T_g (μs)	Bias (%)	1-σ (%)	$\delta_{(R+A)}$ (ns)	1-σ (%)	$\delta_{(A)}$ (ns)	1-σ (%)	$\delta_{(Combined)}$ (ns)	1-σ (%)	$\delta_{(R+A)}/\delta_A$
2	-0.053	0.038	91.5	1.7	93.2	1.6	92.3	1.2	0.982
4	0.021	0.022	109.4	1.1	105.8	1.3	107.6	0.9	1.034
8	0.003	0.016	110.9	1.4	110.0	1.2	110.5	0.9	1.008
16	0.010	0.011	116.5	1.2	116.6	1.4	116.6	0.9	0.999
32	0.0096	0.0086	116.5	1.3	115.6	1.5	116.1	1.0	1.008
64	0.0131	0.0049	114.7	1.2	115.2	1.4	114.9	0.9	0.996
128	0.0152	0.0039	113.7	2.6	114.7	2.6	114.2	1.9	0.991
256	0.0029	0.0036	106.7	3.1	109.6	3.9	108.2	2.5	0.974

There seems to be a persistent small positive bias at the 0.01 % level (Figure 2) – possibly due to some unknown non-ideal behavior; perhaps a small fraction of events exhibit double pulsing – which would require investigation using equipment not available during the campaign. More likely there is a small correlated neutron contribution from the source due to induced fissions in ^{241}Am [13].

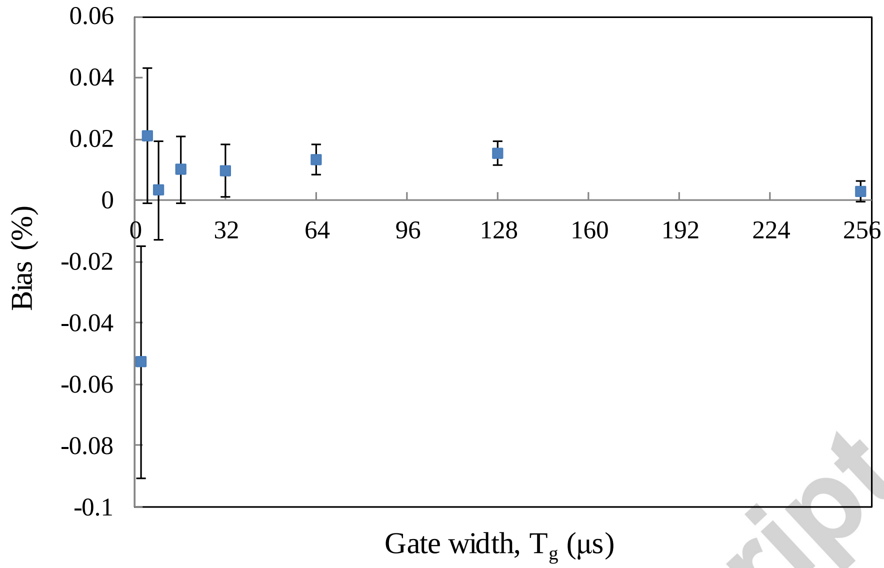

Figure 2. The bias percent in the (R+A)- and A-gates as a function of gate width.

However, the effective dead time parameter estimated from the two histograms are in agreement within counting precision and so we present the plot of the combined results (Figure 3). Statistical uncertainties are quoted at the 1-σ level (68.3 % confidence interval).

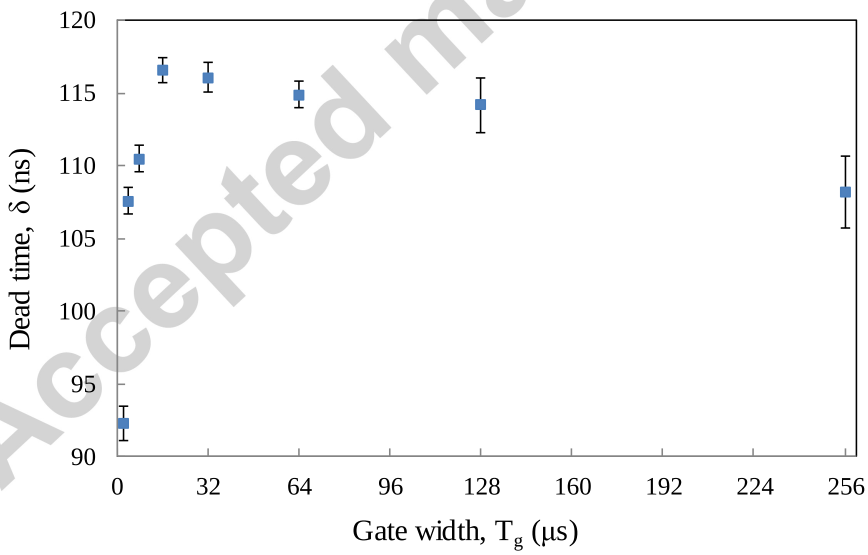

Figure 3. The dead time parameter determined from the combined (R+A)- and A-histograms of the neutron count distribution as a function of gate width.

Shown in Fig.4 is the combined ((R+A)- and A-histogram) dead time parameter extracted using the two alternative expressions which make use of the third and fourth factorial moments of the count distribution, respectively. Despite the large initial uncertainty in the 4^{th} factorial moment results, all three estimates of the dead time are in good accord, which suggests that the model assumptions and approximations are appropriate.

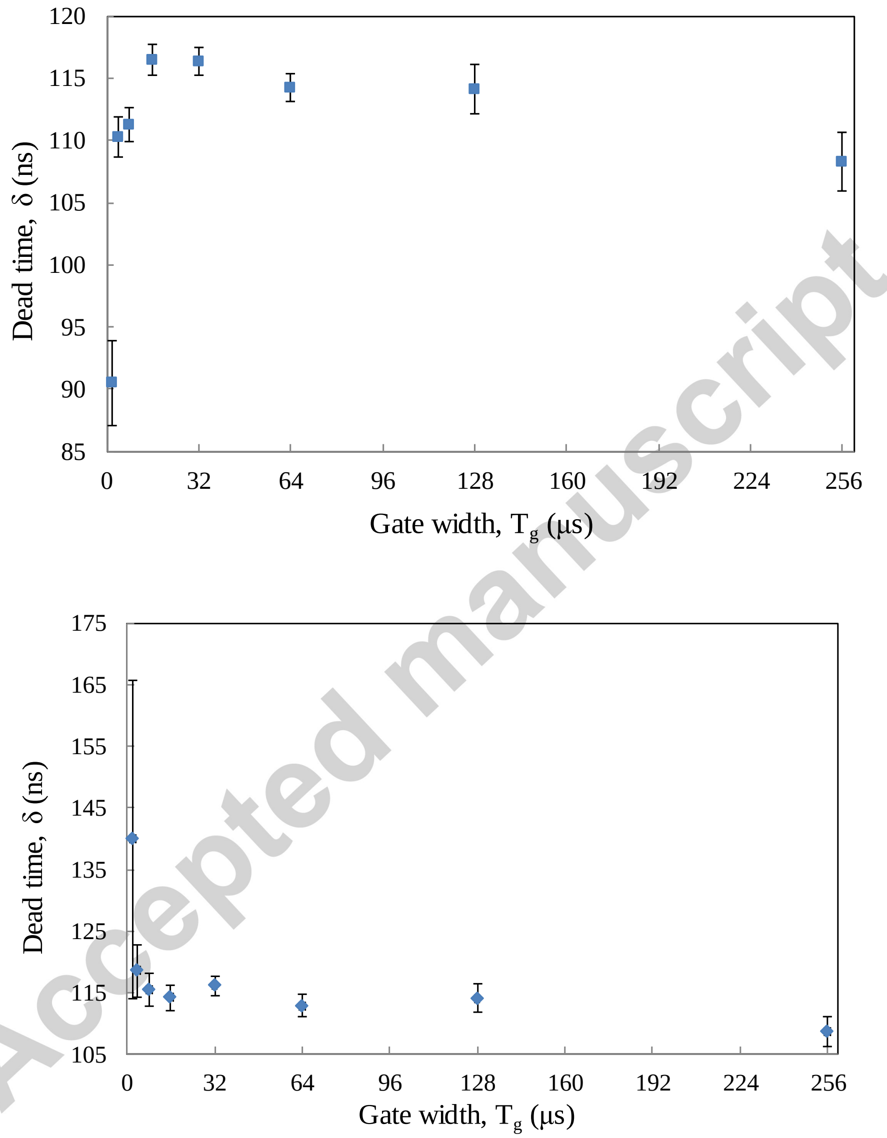

Figure 4. Top: Dead time parameter using the 3rd factorial moment of the combined (R+A)- and A-neutron count distribution. Bottom: Dead time parameter using the 4th factorial moment.

Conclusions and Discussion

It is understood that the recorded number distribution can be decomposed into its reduced factorial moments which provides an alternative unique description of its statistical properties. In this work, for the first time, we use the higher order moments to experimentally explore the effective system dead time. A brief review of the expression used to experimentally estimate effective system dead time from count distributions with a random neutron source, for use in characterizing neutron correlation counters, has been given. Within the theoretical framework of the one-channel paralyzable dead time model, a simple exact expression for the dead time exists in terms of the mean and variance of the count distribution and the width of the counting gate used. We experimentally studied the variation of the dead time parameter estimated as a function of gate width for the ORNL LV-AWCC using AmLi sources.

One might expect the underlying effective system dead time parameter to be a system constant, independent of gate width, characteristic only of the distribution of signal widths formed in the ^{3}He proportional counters. However, this is not what we found in the case of the LV-AWCC. Based on the second factorial over mean squared expression, the dead time peaks at a gate width of about 16 μs. The values at 2 μs and 256 μs are 79 % and 93 % of the 16 μs value, respectively, with the drop on the wide gate side being approximately linear. These variations are large in relation to the measurement precision estimated by statistical analysis and so exhibit some genuine trend. Using a dead time parameter specific to the gate width is not a practical obstacle being readily accommodated in the current generation of dead time correction formulations [14, 15].

An almost identical picture is obtained when using the third factorial moment over mean cubed expression. The experimental uncertainties are somewhat larger (although generally less than by a factor of two) but the overall trend seen using the second moment expressions is closely replicated. When using the fourth factorial moment expression, the uncertainties increase again. For the 2 μs data, the uncertainty is significantly larger; but this is not surprising because the count distribution only extends to $i = 4$, and then with a low probability, renders the higher moment calculation less reliable. Overall, however, the estimates of the effective system dead time obtained remain in excellent agreement, within the estimated precisions, of the other two approaches.

It is not possible from the type of experiment performed here to determine the reason for the observed trend of δ with T_g, but we can speculate several potential reasons as to why the dead time model may not exactly match with reality. Notably, the dead time model assumes a single counting chain and a fixed dead time of the paralyzable type. In practice, the system comprises 8 counting chains and the dead time of each channel follows a distribution corresponding to the variation in time the ^{3}He proportional counter signals spend above threshold. Also, the finite width of the TTL logic pulses and the nature of the pulse train summing process introduces an additional dead time influence which might make the overall system perform in a paralyzable dominant, but not wholly paralyzable, way. And finally, there may be other non-ideal detector behaviors such as double pulsing and baseline shifts in the amplifiers following each event. It is also important to recall that the theory assumes that the dead time is less than the gate width. In the short gate width regime (a few μs) this may not be a wholly valid assumption for the individual channels of the LV-AWCC; since the overall pulse train is a sum of 8 such channels, it seems reasonable to anticipate some deviation from the simplified ideal behavior in this extreme. On the other hand, it is reassuring to find that the dead time parameter extracted using the three different expressions presented paint the same general picture. This suggests that the results are robust and self-consistent at the pragmatic working level.

To illustrate just one way in which the actual LV-AWCC differs from the mathematical model being used, recall that the measured system count rate is the sum of 8 channels: $S_m = S_{m1} + S_{m2} + \ldots + S_{m8}$. If channels 1-4 are those of the inner ring and channels 5-8 those of the outer ring then we can write, for the average dead time free (or dead time corrected) behavior: $S_c = 4\left(\frac{\varepsilon_I/\varepsilon_T}{4/8}\frac{S_c}{8}\right) + 4\left(\frac{\varepsilon_O/\varepsilon_T}{4/8}\frac{S_c}{8}\right)$, where ε_I, ε_O, and $\varepsilon_T = \varepsilon_I + \varepsilon_O$ are the inner-ring, outer-ring and total-system detection efficiencies, respectively. Because ε_I and ε_O are not equal, the preampliers in the inner and outer rings are not equally loaded, and this has a consequence for the overall system dead time behavior as we shall now show. If each channel is subject to paralyzable dead time, δ_o, then for a random neutron source we would expect: $S_m = S_c\left[\left(\frac{\varepsilon_I}{\varepsilon_T}\right)\exp\left(-2\frac{\varepsilon_I}{\varepsilon_T}\frac{\delta_o}{8}S_c\right) + \left(\frac{\varepsilon_O}{\varepsilon_T}\right)\exp\left(-2\frac{\varepsilon_O}{\varepsilon_T}\frac{\delta_o}{8}S_c\right)\right]$. Only when ε_I and ε_O are equal does this expression simplify to the exponential form of the single channel model: $S_m = S_c\exp\left(-\frac{\delta_o}{8}S_c\right)$. For all other cases we see that what emerges in the practical setting is an effective system dead time that partially compensates for the model mismatch. In the present example, in the case of singles counting in the low deadtime correction limit, we see by expansion and comparison of terms that an effective system dead time is: $\delta \approx 2[(\varepsilon_I/\varepsilon_T)^2 + (\varepsilon_O/\varepsilon_T)^2]\frac{\delta_o}{8}$.

Throughout this paper we have emphasized that the analytical dead time correction models are applied with effective dead time parameters. By using a simple model and effective parameters, adequate corrections can often be made provided the corrections are modest. One way of minimizing dead time in future systems is to pay special attention to the proportional counter/amplifier combination and use one amplifier per proportional counter. This shifts the upper dynamic range and for some applications the benefit justifies the additional cost. But there will always be challenge cases where improved dead time treatment is also needed. At the present time all dead time correction approaches used in applied neutron correlation counting are either empirical or are based on simple one-channel models and use effective dead time model parameters.

The value of the present work is that the concept of an effective dead time parameter has been shown by direct experiment to be only an approximation when applied to a representative measurement instrument. The potential systematic error in the dead time parameter determination from the count distribution evidently far exceeds the readily achieved measurement precision reported. But for modest rates, acceptable dead time correction factors are still anticipated and are suitable for practical applications such as international nuclear safeguards. We caution against drawing overly general or sweeping conclusions from the results reported here for a particular instrument, when the main purpose was to introduce and demonstrate a new approach to dead time estimation for practical applications. However, we provide an additional data set and remarks in the Annex which further indicates that dead time behaviors in real systems are more subtle than treated when scrutinized in detail. To explore the underlying behavior in greater detail, we plan to investigate the time domain using list mode data acquisition and to explore more sophisticated dead time models using Monte Carlo simulation of the entire counting system.

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ANNEX

Measurements on the LV-AWCC suggested that the effective dead time parameter varies in a non-monotonic way with gate width. We found this trend to be repeatable and so it cannot be discounted as a consequence of counting precision or chance. Because it is usual practice to operate neutron correlation counters with fixed predelay and coincidence gate settings, the effective dead time is usually only determined at operating condition. We have looked at several counters of commonly used design and found systematic behaviors – i.e. the effective system dead time is not constant. Pending a more complete review than can be offered here of our findings, we present the results for just one additional example: a variant of the Type-I Neutron Coincidence Collar (UNCL-I) [16] in the four slab configuration suitable for the verification of fresh mixed U/Pu oxide (MOX) fuel assemblies by passive neutron coincidence counting. Each slab contains six ^{3}He proportional counters (4 atm partial pressure) and one Amptek 111-A JAB-01 amplifier/discriminator. The 50 ns TTL logic-pulse outputs from each slab is summed in a simple OR daisy chain circuit.

Dead time data were taken using the four similar AmLi sources distributed at the center of the mid-plane of the measurement cavity. All four channels were connected to a PTR-32HV [17] list mode data acquisition platform, but only the results for the pulse train from the summed detector response will be presented here. As previously noted, list mode data acquisition is an excellent option to perform this type of analysis as a single pulse train can be reanalyzed in post-analysis for various predelay and gate width values without having to retake a measurement. But, because only a single data set (file of intervals between recorded events) is used, the numerical results of dead time will of be course correlated (i.e. there will be a non-zero covariance between them). We used the INCC export function within the evaluation menu in PTR-32HV to create files in a format that is commonly used in the international safeguards community. Twenty-four cycles of 300 seconds were acquired. The overall counting rate was approximately 45,000 cps nearly evenly distributed between the four amplifiers. The standard 4.5 μs predelay and 64 μs gate width timing windows were selected at the time of acquisition and the files were analyzed offline for a range of gates. The resulting dead times and biases are reported in the Table A1. Plots of the key results are also provided (Figures A1-A4).

For short gate widths, less than about 16 μs, we see some evidence of significant bias, suggesting that the standard (recommended) 4.5 μs predelay does not completely allow the system to recover following an event. We again find good agreement between the dead time estimates derived from the second, third and fourth reduced factorial moments. Below about 16 μs, the dead time values obtained begin to trend with gate width to a degree which is significant compared with the estimated counting precision. Note we are presenting the dead time results for the combined (R+A)- and A- histograms. In light of the apparent bias at short gate widths, one might suppose that the dead time values at short gates extracted solely from the (R+A)-number distribution would be strongly different that that obtained using only the A-distribution. However, this is not the case. Both results exhibit similar trends as a function of gate width even though the predelay for the A-number distribution is of the order of 1000 times longer.

Table A1. Results of dead time analysis for (R+A)- and A-gates, including bias

T_g (μs)	Bias (%)	1-σ (%)	$\delta_{(R+A)}$ (ns)	1-σ (%)	$\delta_{(A)}$ (ns)	1-σ (%)	$\delta_{(Combined)}$ (ns)	1-σ (%)	$\delta_{(R+A)}/\delta_A$
2	0.044	0.038	142.8	1.5	137.1	1.4	140.0	1.1	1.04
4	0.025	0.020	155.9	1.4	153.2	1.8	154.6	1.1	1.02
8	0.018	0.018	160.8	1.5	158.8	1.7	159.8	1.1	1.01
16	0.0031	0.012	163.0	1.6	162.2	1.5	162.6	1.1	1.01
32	0.0018	0.0073	162.9	1.4	162.6	1.8	162.8	1.1	1.00
64	0.0028	0.0047	164.8	2.1	163.5	2.2	164.1	1.5	1.01
128	0.0040	0.0034	163.9	2.7	162.9	2.8	163.4	1.9	1.01
256	0.0014	0.0022	165.2	2.8	165.2	3.3	165.2	2.2	1.00

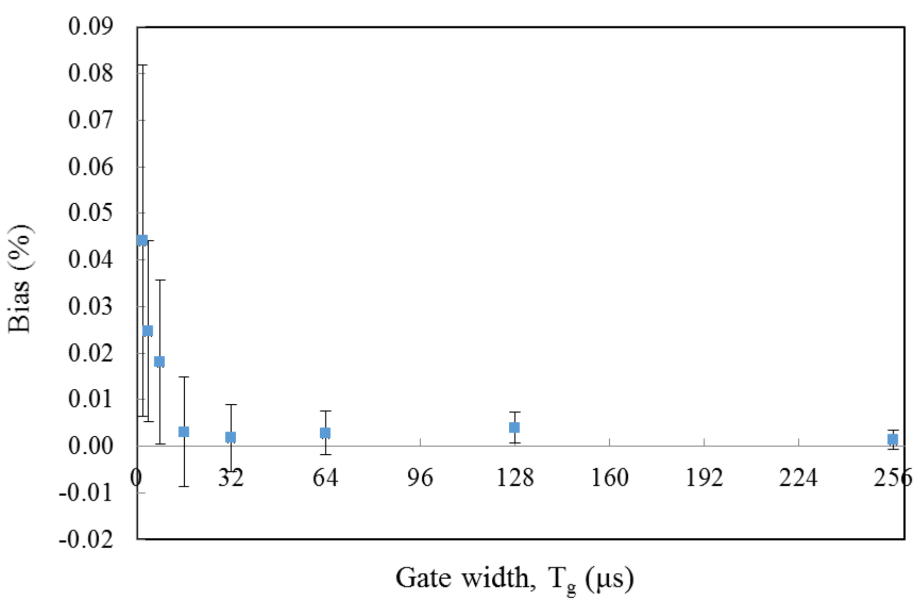

Figure A1. The bias percent in the (R+A)- and A-gates as a function of gate width.

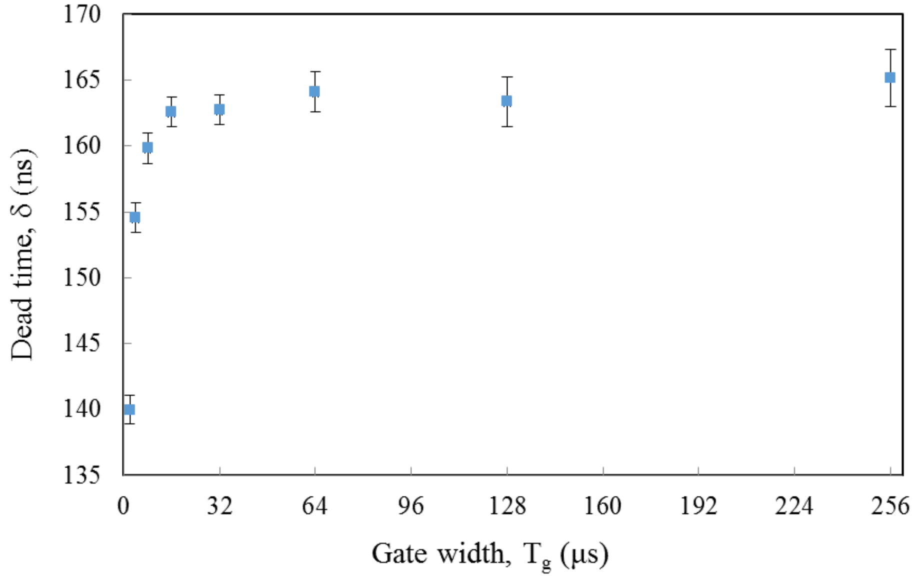

Figure A2. The dead time parameter determined using the 2nd factorial moment of the combined (R+A)- and A-histograms of the neutron count distribution as a function of gate width

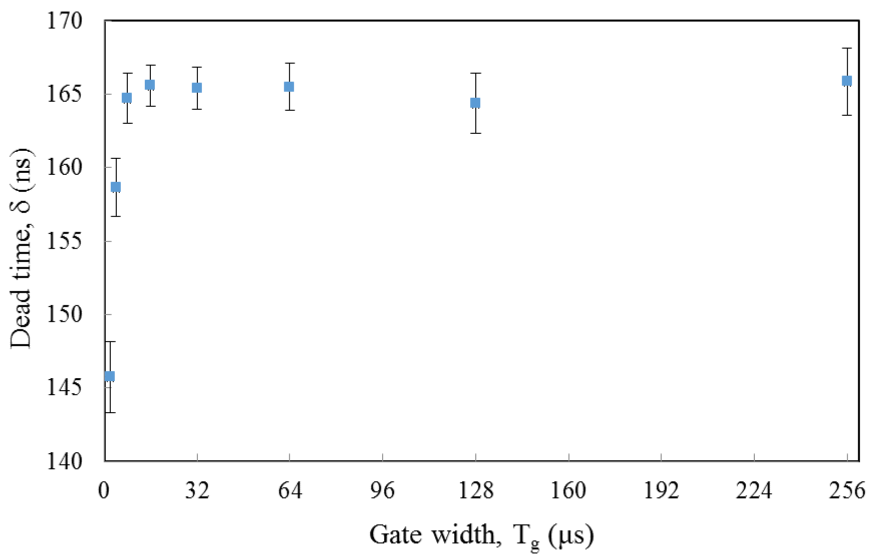

Figure A3. The dead time parameter using the 3rd factorial moment of the combined (R+A)- and A-neutron count distribution.

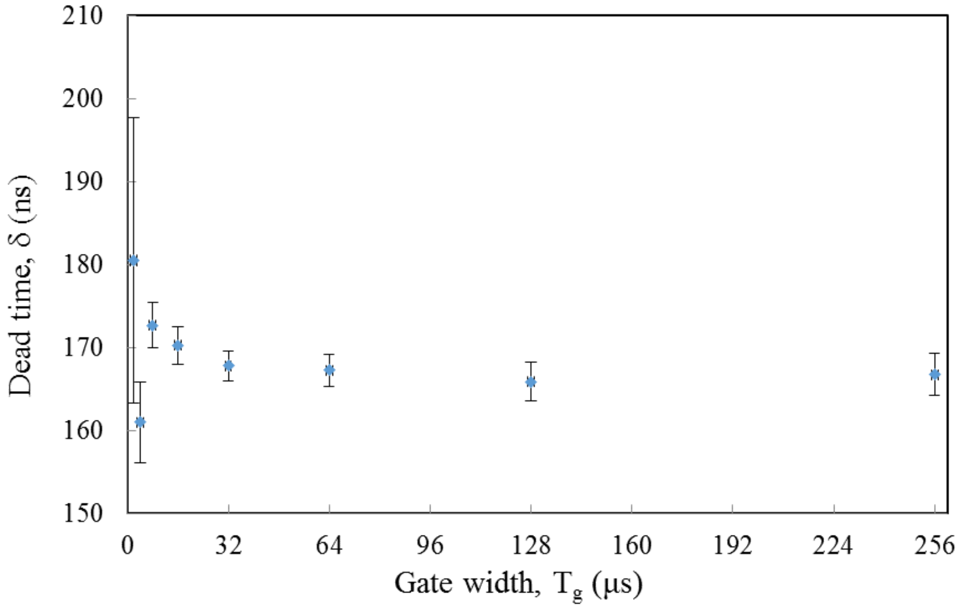

Figure A4. The dead time parameter using the 4^th^ factorial moment of the combined (R+A)- and A-neutron count distribution.

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